Mathematics Methods and Thermodynamics

Homework Help - Integral Range Transform

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September 11, 2017

- To answer the question on Fourier series when the range is not equal to $-\pi$ to π you need to do change the integral range
- To do this we must apply a coordinate transform, $x = G(u)$

$$\int_{a}^{b} F(x)\; dx = \int_{-\pi}^{\pi} F(G(u))\; du$$

- The goal is to identify what $G(u)$ is

- Lets consider the general case

$$\int_a^b F(x)\,dx = \int_c^d F\left(G\left(u\right)\right)\,du$$

- If $|b-a|$ is the same as $|d-c|$ then you can just shift your function in the usual way.

$$F(x) \rightarrow F(x-A)$$

- Where A is the amount you need to shift by.
- The above notation can be a little misleading since $x \rightarrow x - A$, instead it would be better to treat this as a coordinate transformation $x = u - A$

- At the lower limit $x = a$ and we want $u = c$
- This gives us $x = u + A \rightarrow a = c + A$
- You can work out A by solving this expression for A

$$c + A = a$$

$$A = a - c$$

- If $|b-a|$ is the NOT the same as $|d-c|$ then you also need to apply a scaling factor.

$$F(x) \rightarrow F\left(Au + B\right)$$

- Now we work out the unknowns simultaneously from our two know limit relations
- At the lower limit

$$x = Au + B \rightarrow a = cA + B$$

- At the upper limit

$$x = Au + B \rightarrow c = dA + B$$

- Solving for B

- After working out A we work out A in a similar manner but using the upper limits rather than the lower

$$b = dA + B$$

$$A = \frac{b - B}{d}$$

- Plugging in A

$$A = \frac{b - (a - cA)}{d}$$

$$A = \frac{a - b}{c - d}$$

- So A is just the ratio between the ranges a to b and c to d

$$A = \frac{a-b}{c-d}$$

- Returning to B we can now plug in A to get

$$A = \frac{ad-bc}{c-d}$$

- So for a general change of integral ranges from

$$\int_a^b F(x)\,dx = \int_c^d F(Au + B)\,dx$$

- We need the following coordinate change

$$x = Au + B$$

$$A = \frac{a - b}{c - d}$$

$$B = \frac{ad - bc}{c - d}$$

- Since this is a coordinate change we also need to consider how the size of an infinitesimally small quanta changes

$$\int_a^b F(x)\,dx = \int_c^d F(Au+B)\frac{duA+B}{du}\,du$$

$$\frac{duA+B}{du} = A$$

$$\int_a^b F(x)\,dx = \int_c^d F(Au+B)A\,du$$

Summary

- To summarise

$$\int_a^b F(x)\,dx = \int_c^d F(Au+B)A\,du$$

- Where we have the following

$$A = \frac{a-b}{c-d}$$

$$B = \frac{ad-bc}{c-d}$$