Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name and section: _____

Instructors name: _____

1. What do differentials represent? [1]

2. List 4 ways to represent differentials of $f(x)$ [2]

3. Differentiate the following [1]

$$\frac{d}{dx}\left(3x^3 - \frac{1}{2}x^2\right)$$

4. [2]

$$\frac{d}{dx}\left(10^{3x}\right)$$

5. [2]

$$\frac{d}{dx}\left(7^{x^2}\right)$$

6. [1]

$$\frac{d}{dx}\left(\mathrm{e}^{-x}\right)$$

7. [2]

$$\frac{d}{dx}\left(\frac{\mathrm{e}^{a}}{x^2-1}\right)$$

8. [2]

$$\frac{d}{dx}\left(\mathrm{e}^{a}\ln\left(x^2-1\right)\right)$$

9. [2]

$$\frac{d}{dx}\left(\cos\left(3x+x^2\right)\right)$$

10. [3]

$$\frac{d}{dx}\left((2x-1)\cos\left(x^2\right)\right)$$

11. [4]

$$\frac{d}{dx}\left(\frac{2x-1}{x^2+1}\right)$$

12. [4]

$$\frac{d}{dx}\left(2(2+3x^2)(x^3-2x)^2\right)$$

13. [5]

$$\frac{d^3}{dx^2dy}\left(cos(2x^3+y)\right)$$

14. What is the difference between a concave and convex function [1]

15. What are the conditions for a "well behaved" function [1]

16. Give examples of functions that do not satisfy each of the conditions for a "well behaved" functions (one example for each condition, specify which condition it is) [3]

17. How does a Legendre transform change a function, (what is the information represented as) [2]

18. How does a Legendre transform change a function, (what is the information represented as) [2]

19. What are the conditions of a Legendre transform [1]

20. What is the Legendre transform of $f(x) = x\ln\,(x)$, $x > 0$ [5]

21. The Legendre transform of x^2 is $\frac{p^2}{4}$. Show that the Legendre transform is its own inverse [4]

22. Use the Legendre transform to derive a thermodynamic function starting from the internal energy [4]

23. For the thermodynamic function you derived, give the Maxwell relation it provides [4]