Answer the questions on your own paper and hand that it.

Name: _____

Instructors name: _____

1. What are ordinary differential equations, what properties must they have [2]

2. When solving an ODE we must find a function that satisfies the conditions laid out by the ODE. Find an expression for y in these ODEs

   (a) [2]

   $$y' - 2y = 2$$

   (b) [2]

   $$y' - x = 3$$

   (c) [3]

   $$y' - 6xy = 0$$

   (d) [4]

   $$y' - \cos(x)\cos(y) = \cos(y)$$

3. Show that $(1 - x)$ is a solution of [3]

   $$xy'' + (1 - x)y' + \lambda y = 0$$

4. Show that $\left(6x^2 - 2\right)$ is a solution of [3]

   $$\left(1 - x^2\right) y'' - 2xy' + l\,(l + 1)\, y = 0$$

5. Show that $\left(x^2 - 1\right)$ is a solution of [3]

   $$y'' - 2xy' + \lambda y = 0$$

6. What is an inexact differential and why are they hard to solve when they appear in ODEs [2]

7. What is a Fourier series? How is it different from a Fourier transform [2]

8. Compute the first three coefficients of the Fourier series for the following functions between $-\pi$ and π

(a) [3]

$$x$$

(b) [3]

$$x^2$$

(c) [3]

$$\mathrm{e}^{-x}$$

9. Compute the first three coefficients of the Fourier series for the following functions between 0 and 3

(a) [2]

$$x$$

(b) [2]

$$x^2$$

(c) [2]

$$\mathrm{e}^{-x}$$

10. How would you compute the error between your Fourier series and the exact function? [2]